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Bridging Engineering and Economics

Since 1973

Cost Risk Allocation

Objectives, Tendencies and Limitations

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Overview

- **Overview**
  2-3 minutes
- **What Is Cost Risk Allocation?**
  10-15 minutes
- **Defining The Threat**
  10-15 minutes
- **Minimizing Average Budget Overrun**
  15-20 minutes
- **Minimizing Budget Overrun Semi-Varian**
  20-30 minutes
- **In Conclusion**
  2-3 minutes

Proverb

Knowledge is better than blind practice.

-Fortune Cookie

Lucky numbers: 7 9 23 36 41, 19

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What Is Cost Risk Allocation?

A Risk By Any Other Name

$, € vs. %

Cost
*The price paid to acquire, produce,
accomplish, or maintain anyth*
Dictionary.c

Risk
*The possibility of suffering
harm or loss; danger*
an Heritage
Dictionary

Cost Risk Allocation
*A process by which costs of subordinate WBS
elements are allotted such that they sum
to the parent cost at the selected cost risk*
(working definition)

Cost Risk Conseque
*The average additional cost
suffered*
(working definition)

ost Risk
*robability of incurring
additional cost to the budget*
Dictionary.com

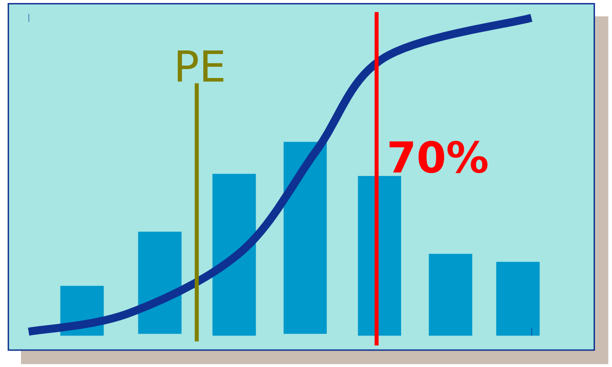
Allocate
o distribute according to a plan; allot
American Heritage Dictionary

Risk Dollars
*of funds needed to bring the
TBE value up to a selected probability level*
AFCAA CRH

Allocate Risk Dollars
To distribute risk dollars back to WBS elements
(paraphrase of presentation title of S. Book)

Uncertainty Is Understanding

- **Point Estimate Has No Context on Its Own**
  - How precise is our model?
  - How likely will we beat the P.E.?
  - What elements drive the uncertainty?
- **Cost Uncertainty Analysis…**
  - Quantifies precision of the model
  - Identifies ranges of likely costs
  - Reveals worrisome elements
- **However, Uncertainty *Doesn't* Add Up**
  - Accountants don't like this fact
  - Managers want an answer that they understand
  - Hard to compare against execution progress
- **Allocated Costs Add Up (just like P.E. and Mean)**
  - More statistically meaningful than point estimate
- **But Beware**
  - Cost risk of elements will change if their cost changes
  - Allocated estimate loses context of model precision
- **But What is a Good Cost Risk Allocation Method?**
  - It ultimately comes down to priorities



Cost = $10,235,329.88

Cost = $10M - $12M

$5.2M + $6.3M = $11.5M

@73% @73% @80%

Priority One

- **First, Define What is Important:** *(may conflict)*
  - Minimizing overruns that may occur
  - Reducing chance of a budget overrun
  - Protecting important systems from failure
  - Meeting schedule demands
  - Identifying money flow problems
  - Tracking well to EVM during execution
  - Etc.
- **Next, Figure Out What You Can Manage:**
  - Identifying and mitigating risk
  - Holding funds in reserve
  - Schedule and scope
  - Etc.
- **And What You Can't Manage:**
  - Due to legal issues (color of money)
  - Due to bureaucracy (approval and reporting)
  - Due to project inertia (contracts and penalties)
  - Etc.

Proverb

Digging a hole in the right place is more important than digging the hole right.

GOOOOAL!!!!!!

- **Our Ultimate Goal Is Project Success**
  - A good start means better chance of success
  - Helps our manager make informed decisions
- **Our Realistic Goal Is Getting WBS to Add**
  - For whatever reason…
    - … we must capture risk dollars in line items
    - … we cannot show a reserve line
- **A Cost Risk Allocation Scheme…**
  - …should reliably optimize what concerns u
- **Cost Risk Allocation Is a Limited Tool**
  - Fails to capture important issues that impact budget viability…
    - … schedule risk, money flow, contract vehicle, risk mitigation, etc.

Proverb

When all you own is a hammer

everything looks like a nail.

everything looks like a nail.

Could our model capture these?

The First Rule of Allocation

Perform cost risk allocation only when the WBS must sum to a budget at a specified cost risk.

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Quick Example

- **Ex: Allocate Air Vehicle for 25% Cost Risk**
  - i.e., 75% probability of being under budget
- **What is the "correct" way?**
  - Semantically correct as long as WBS adds up
- **Compare four methods**
  - (bad) Subtract 5.4 from largest elements
  - (bad) Subtract 1.8 from each element
  - (good) Minimize average size of cost overrun
  - (good) Minimize semi-variance *(explained later)*

Total is 5.4 less than sum of children

Uncertainty Statistics	
WBS/CES	**75.0 %**
Air Vehicle	168.2
Design & Dev.	34.7
Prototypes	18.6
Software	120.3

Σ=173.6

		Four Cost Risk Allocation Methods			
WBS/CES	**Point Estimate**	**Subtract From Largest**	**Subtract 1.8 From Each**	**Average Overrun**	**Overrun Variance**
Air Vehicle	111.5 (32%)	**168.2** (75%)	**168.2** (75%)	**168.2** (75%)	**168.2** (75%)
Design & Dev.	25.0 (25%)	34.7 (75%)	**32.9** (67%)	**34.0** (72%)	**29.9** (54%)
Prototypes	9.7 (20%)	18.6 (75%)	**16.8** (66%)	**18.1** (72%)	**14.6** (54%)
Software	76.8 (41%)	**114.9** (71%)	**118.5** (74%)	**116.1** (72%)	**123.7** (77%)

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Defining The Threat

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Two Camps of Thought

An overrun may be a symptom of project illness

$$threat \propto overru$$

$$threat \propto overrun^2$$

The "Cost Camp"

- **Do You Believe?**
  - Your level of angst increases as overrun increases
  - Subsystems should meet their budget regardless of cost
  - The percentage of overrun defines the threat of failure
  - Allocation should be proportional to the cost risk

The "Variance Camp"

- **Do You Believe?**
  - Your level of angst rapidly accelerates as overrun increases
  - Less costly subsystems are less important to stay within budget
  - The dollar amount of the overrun determines the threat of failure
  - Allocation should be in proportion to the square of the cost risk

The risk of project failure encompasses more than a cost overrun

At The Mall

Proverb

Expenses grow to fill the budget.

Trip To The Mall

You give Ben and Alice each $15 for a CD.
How much change do you get back?

Ben paid $12. Alice needs $2 more.
Did you overrun by $2 or recover $1?

A cost model reports that you get $1 back.
In our world, you need $2 more to succeed.

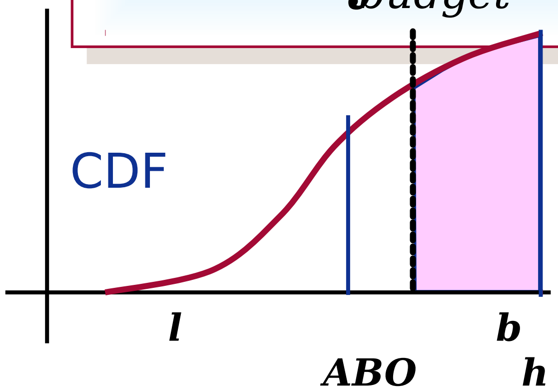
"Overrun" Defined

Proposed Definitions

Average Budget Overrun (ABO)
The cost risk consequence of a budget assigned to an element

$$ABO = \int_{budget}^{\infty} (c - budget)\, f(c)\, dc$$

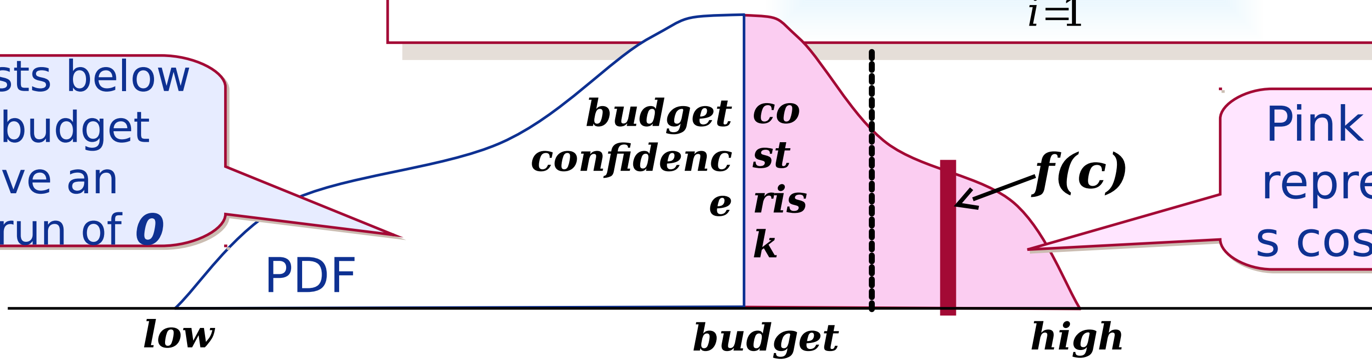
Where,
c is a potential cost
f(c) is the element's PDF



Total Average Budget Overrun (TABO)
The cost risk consequence for a sum of elements given money from under-budget elements cannot be recovered

$$TABO = \sum_{i=1}^{n} ABO_i$$

All costs below the budget have an overrun of ***0***



Pink area represents cost risk

Overrun Defined (cont.)

Proposed Definitions

Budget Overrun Semi-Variance (BOSV)
A measure of risk consequence using the squares of each potential cost risk consequence weighted by probability of occurrence

$$BOSV = v^2 = \int_{budget}^{\infty} (c - budget)^2 f(c)\,du$$

Where,
c is a potential total cost
f(c) is the element's PDF

Total Budget Overrun Semi-Variance (TBOSV)
A measure of risk consequence for a sum of elements including the impact of pairwise correlations among elements

$$TBOSV = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i,j} \sqrt{BOSV_i BOSV_j}$$

Where,
$\rho_{i,j}$ is the correlation between ***i*** and ***j***
ρ represents a full correlation matrix
BOSV is Budget Overrun Semi-Variance

Look familiar? Analogous to variance.

$$\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{i,j}\sigma_i\sigma_j$$

Where,
$\rho_{i,j}$ is the correlation between ***i*** and ***j***
σ^2 represents the variance for ***i***

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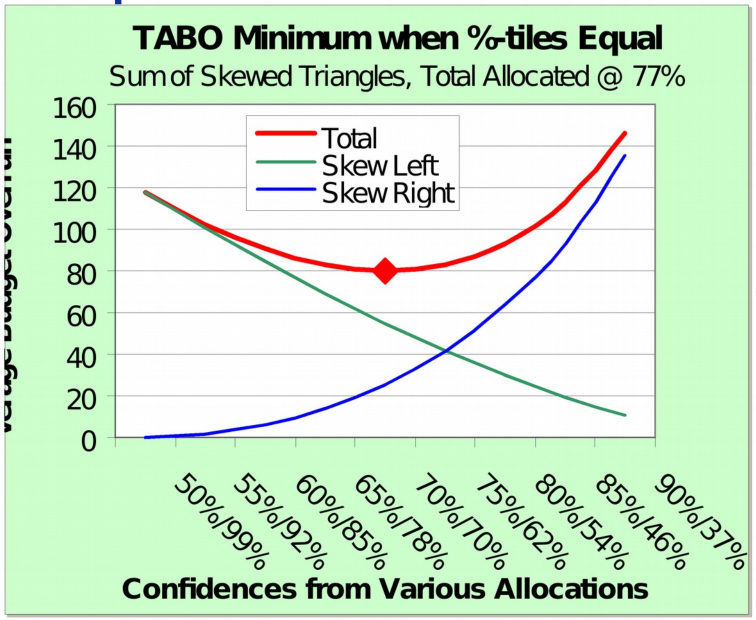
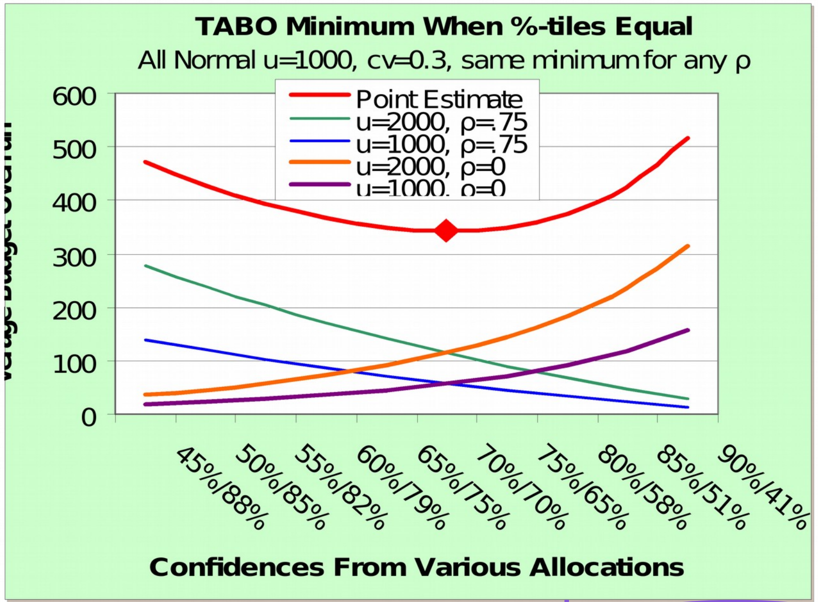
Minimizing Average Cost Overrun

Simulation Results

- **Charts show simulation results for sum of skewed and sum of correlated elements**
  - The total budget was 2370 *(77%-tile)* & 6945 *(81%-tile)* for respective charts, TABO in red
  - The uncertainty levels of A, B were altered by *increments of 5%* and ABO plotted
  - The uncertainty levels for C, D also displayed on X-axis for reference and ABO plotted
  - The total average budget overrun was plotted for each pair of element confidence levels
- **Result: Total average budget overrun was minimum when element confidences were equal**





WBS	Distribution	Low	Mode	High	Allocated	Alloc %-Tile
Total					2370	77%
Skew Left	Triangular	1000	2000	2000	1450	70%
Skew Right	Triangular	1000	1000	2000	920	70%

WBS	Distribution	Mean	CV	Allocated	Alloc %-Tile
Total		4000		6945	81%
A (Cor w/ B)	Normal	1000	0.2	2315	70%
B (Cor w/ A)	Normal	1000	0.2	1158	70%
C (Ind.)	Normal	1000	0.2	2315	70%
D (Ind.)	Normal	1000	0.2	1158	70%

Peanut Butter

Optimizing the "Cost Camp" Way…

- **When *Allocating* to Minimize the Total's *Average Budget Overrun*…**
  - …everything is already captured in the uncertainty statistics…
  - …so don't worry about integrating additional measures into method

Negative Correlatio n?

Minimal Total Average Budget Overrun

Allocate so that all elements receiving funds end up at the same confidence level.

i.e. move money around WBS

- **The Only Decisions to Make Are…**
  - Where to allocate from – this should be where you can manage funds
  - Where to allocate to – usually the lowest level WBS you are reporting
    - Also reasonable to allocate to immediate children and work up the WBS
  - How precise to be with uncertainty levels *(why is explained later)*
    - About ±1% is fine – after that round and report

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Allocation Destination

- **Determine…**
  - … WBS detail to report
  - … Where to allocate from
    - i.e., where you manage funds
  - … Where to allocate to
    - i.e., who is adjusted

Multi-Tier Allocation Options:

- **1) Allocate to lowest WBS**
  - And then sum up WBS
  - ↑ One step process is easier to implement
  - ↓ Mid-WBS values change if level of detail changes
- **2) Allocate down WBS**
  - Allocate from total to immediate children
  - And then, allocate from child to its grandchildren, etc.
  - ↑ Keeps values consistent if report detail changes
  - ↓ More steps to perform

If Funds Managed At R&D, Prod & O&S Allocate 3 Times

Total	850
R&D	130
R	50
D	80
Prod	575
Non-Rec	125
Rec	450
O&S	145
O	90
S	55

Examples Show Funds Managed At Total

Total	850
R&D	126
R	46
D	80
Prod	524
Non-Rec	128
Rec	456
O&S	145
O	90
S	148

TABO Calculation

Adjusting Once for Tot. Ave. Budget Overrun:

(Easy to do and results close to optimal)

$$delta = budget_{total} - \sum_{i=1}^{n} cost_i$$

$$budget_i = cost_i + delta \frac{\sigma_i}{\sum_{j=1}^{n} \sigma_j}$$

Where,

budget$_{total}$ is the target cost for total for desired cost risk

cost$_i$ is row ***i***'s cost with the same cost risk as ***budget***$_{total}$

delta is the amount to distribute among rows

budget$_i$ is the new, adjusted cost for row ***i*** after allocation

σ_i is the standard deviation of row ***i***

Replace σ for the *square root of BOSV* if you feel like calculating it

Recursive Formula:

(For penny pinchers)

$$pct_0 = F_T(budget_{total})$$

$$cost_{r,i} = F_r^{-1}(pct_i)$$

$$delta_i = budget_{total} - \sum_{r=1}^{n} cost_{r,i}$$

$$budget_{r,i+1} = cost_{r,i} + \frac{delta_i \sigma_r}{\sum_{j=1}^{n} \sigma_j}$$

$$pct_{i+1} = \frac{1}{n} \sum_{r=1}^{n} F_r(budget_{r,i+1})$$

Where,

budget$_{total}$ is the desired total cost

pct$_i$ is percentile of the rows to sum

delta$_i$ is the amount to distribute

cost$_{r,i}$ is the cost for row ***r*** at ***conf***$_i$

$\boldsymbol{\sigma_r}$ is the standard deviation of row ***r***

F$_r$***(v)*** is the CDF for row ***r***

TABO Example

Calculation Example When Allocating to Lowest Reported WBS Level:

- Step 1: Pick cost risk of 25% (75%-tile)= $608.94M *(this assumes we manage funds at total)*
- Step 2: Choose where to allocate to… 3rd level WBS elements *(lowest reported level)*
- Step 3: Calculate ***delta***: Sum at @ 75% = 625.98 - ***budget*** = -17.04
- Step 4: Prorate ***delta*** for each element weighted by standard deviation *(or TABO)*
- Step 5: Determine confidence levels for each element's cost
- Step 6: If percentiles aren't close enough, use the weighted mean of the new levels as your next percentile, pct_{i+1}, and then return to step 3. *(Twice through is sufficient)*

WBS/CES	75% -Tile	Std Dev	Calculate Adjustment	Allocated	New %-Tiles
Total ($M)	*$608.94*			**$608.94**	75.0%
Procurement	*$385.66*			**$393.92**	75.2%
Manufacturing (Air Force)	$272.67	$ 68.80	-17 * 69 / 188 = -6.24	**$266.43**	**72.2%**
Ground Station LRIP Support	$0.88	$ 0.25	-17 * 0.25 / 188 = -0.02	**$0.86**	**72.7%**
Transportation (AF)	$2.00	$ 0.57	-17 * 0.57 / 188 = -0.05	**$1.95**	**72.7%**
Manufacturing (Army)	$125.87	$ 29.59	-17 * 29 / 188 = -2.68	**$123.19**	**72.2%**
Transportable Ground Stations	$0.91	$ 0.24	-17 * 0.24 / 188 = -0.02	**$0.89**	**72.5%**
Transportation (Army)	$0.60	$0.00	-17 * 0 / 188 = -0.00	**$0.60**	

Close Enough

TABO Step Down

Calculation Example when Stepping Down WBS:

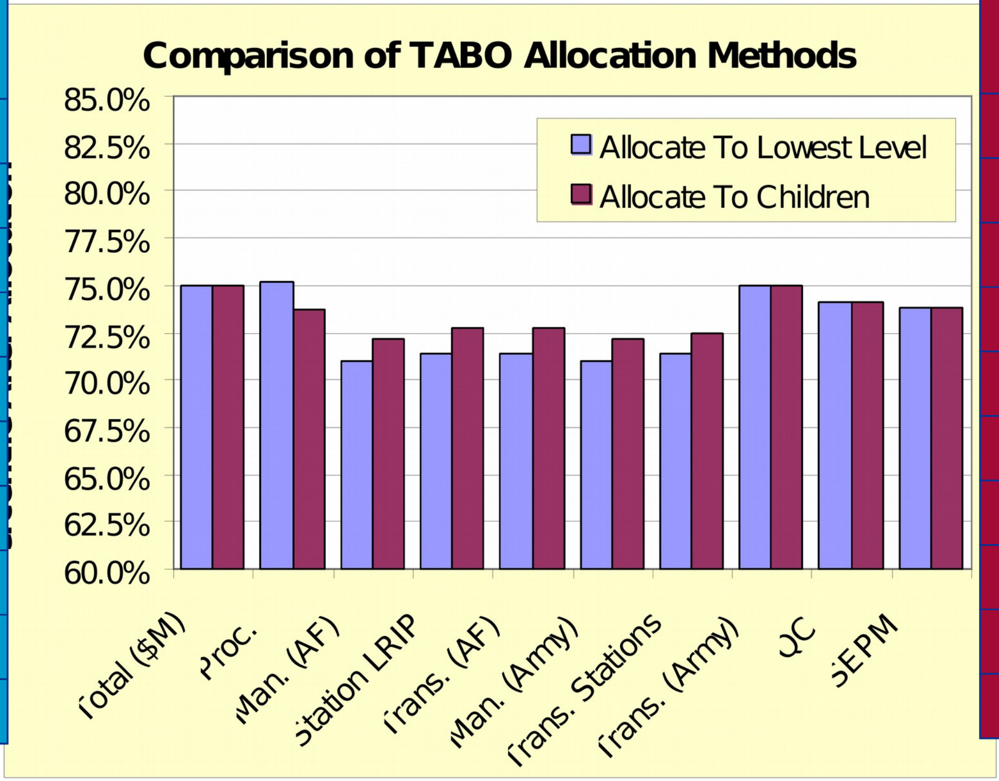
- Step 1: Pick project budget = $608.9M (75% percentile)
- Step 2: Allocate budget for 1st Level to 2nd level WBS items *(its immediate children)*
  - Step 2-1: Calculate $\boldsymbol{delta_1}$: ***budget*** of 608.9 - Sum at @ 75% of 616.6 = -7.7
  - Step 2-2: Prorate $\boldsymbol{delta_1}$ for each element weighted by standard deviation (or TABO)
  - Step 2-3: If percentiles aren't close enough, use weight mean of percentiles and repeat step 2
- Step 3: Take allocated budget for each 2nd level WBS element and allocate to 3rd level
  - Repeat steps 2-1 through 2-3 for each 2nd level $\boldsymbol{budget_2}$, using $\boldsymbol{budget_2}$ - sum of children @73.7%
- Step 4: If report contains 4th+ level WBS, Repeat step 3 for elements @ each level

WBS/CES	75% -Tile	Std Dev	2nd Level WBS Adjustment	Apply to 2nd Level	%-Tiles	3nd Level WBS Adjustment	Allocated	%-Tiles
Total ($M)	*$608.9*			**$608.9**	**75.0%**		**$608.9**	75.0%
Procurement	*$393.5*	**$86.0**	-7.7*86/174= -3.8	**$389.7**	**73.7%**		**$389.7**	73.7%
Manufacturing (AF)	$272.7	$ 68.8		*$269.8*	*73.7%*	-8.9*69/99= -6.2	**$263.6**	**71.0%**
Ground Station LRIP	$0.88	$ 0.3		*$0.87*	*73.7%*	-8.9*0.3/99= -0.02	**$0.85**	**71.4%**
Transportation (AF)	$2.00	$ 0.6		*$1.97*	*73.7%*	-8.9*0.6/99= -0.05	**$1.92**	**71.4%**

TABO Side By Side

Cost Risk Allocation
To Lowest WBS Level

Allocated	%-Tiles
$608.9	75.0%
$389.7	75.2%
$263.6	71.0%
$0.85	71.4%
$1.92	71.4%
$121.8	71.0%
$0.88	71.4%
$0.60	
$10.6	74.1%
$208.6	73.8%



Cost Risk Allocation
To Immediate Children

Allocated	%-Tiles
$608.94	75.0%
$393.92	73.7%
$266.43	72.2%
$0.86	72.7%
$1.95	72.7%
$123.19	72.2%
$0.89	72.5%
$0.60	
$10.39	74.1%
$204.63	73.8%

Lather, Rinse, Repeat?

Proverb

To err is human, to measure it divine.

How Many Times Should We Iterate?

- **Once, usually; otherwise, twice**
  - Don't sweat the 0.001%-tile of confidence!
  - Too much precision is misleading…
  - If you allocate to the penny, it implies the estimate is very precise.
  - *Example: Guess the precision of these estimates: $254,359.25 **vs.** $250,000*
- **I *humbly* suggest that two values are essentially the same…**
  - …at the 2nd significant figure of standard deviation or 1% of confidence
- **I *humbly* suggest that you round at the second digit of standard deviation***
  - *Example: Cost is $**235**9.25, σ is $**23**8.77… thus, report $**236**0 with σ of $**24**0*
  - Or round at first digit of the difference between values 1% confidence apart
  - *Example: Cost is $**268**.36 @73%-tile and $**265**.95 @ 72%-tile…*
    - *…difference is **2**.38… thus, report cost @73%-tile as $**268***
  - Rounding at these positions retains an extra digit of padding for precision

My Suggestion for Rounding

Round to the 2nd digit of the deviation after all intermediate calculations are complete.

* Examples at NIST Physical Constants web site[2]

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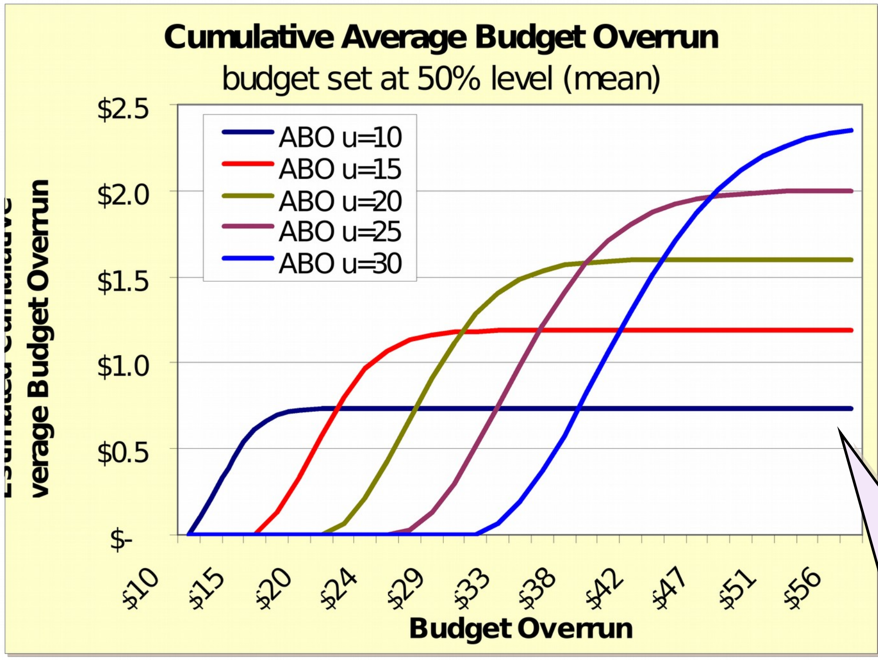
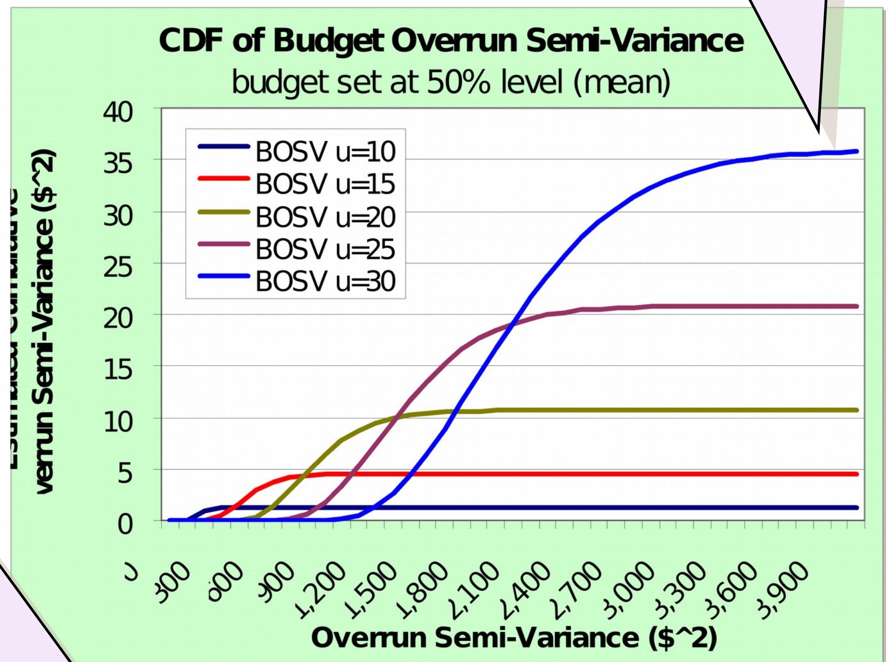
Minimizing Overrun Semi-Variance

TABO vs. TBOSV

- **Total Budget Overrun Semi-Variance Is *Tough to Visualiz***
  - The charts below compare “Average” vs. “Semi-Variance”
  - “average overrun” progresses linearly as mean increases
  - “overrun variance” progresses at a rate of R^2 as overrun increases

3x the cost receives 9x the weighting





Mean	SD
$10	$3
$15	$5
$20	$6
$25	$8
$30	$9

Heights represent element weightings

Mean	SD
$10	$3
$15	$5
$20	$6
$25	$8
$30	$9

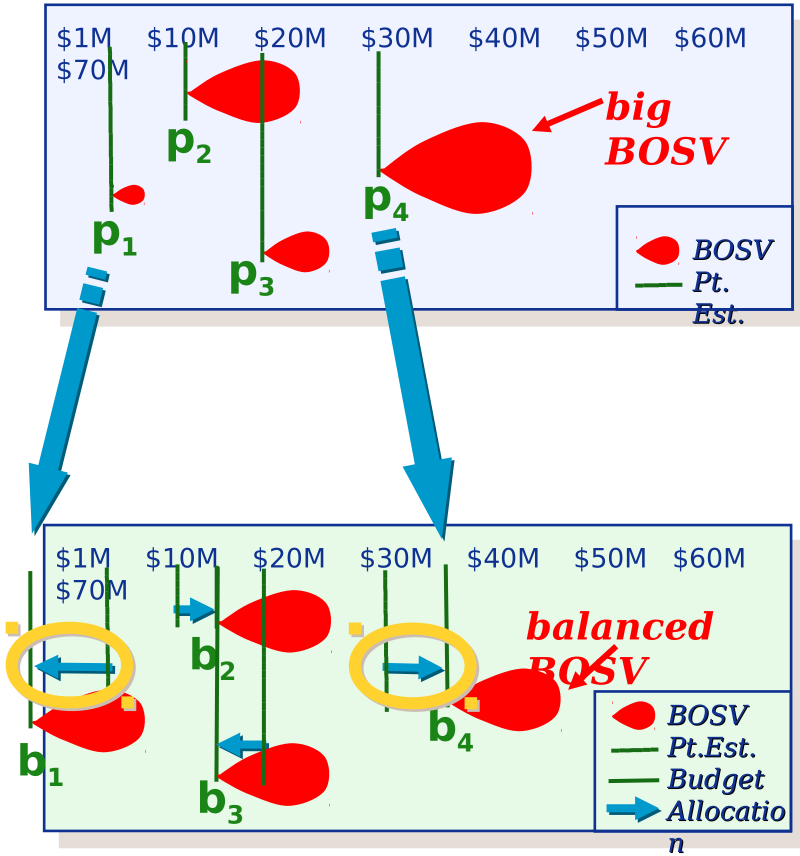
Overrun Semi-Variance

Optimizing the "Variance Camp" Way…

- **Elements with large risk consequence have disproportionate importance**
  - Argument goes that we should protect them
  - Since we are dealing with variance, we must take correlation into account
- **Behavior of Optimal Solution**
  - A budget, $\boldsymbol{B}$, such that the total budget overrun semi-variance (TBOSV), $\boldsymbol{v}^2_{total}$, is minimal.
  - All things being equal, we would want equal $\boldsymbol{v}_k$
  - Desired BOSV decreases as element's correlation to other elements increases
  - The method begins to resemble "cost camp" method as more and more correlations increase

Unfortunately…

- **The Optimal Solution is Not Viable**
  - Elements with small BOSV could move dramatically–potentially outside valid bounds
  - Tough math to solve, too.
- **How to Stay Within Distribution Bounds?**
  - Put limits on elements' ranges of movement
  - Or, we can "anchor" our solution to something
    - Larger variances move further from "anchor"
    - Smaller variances remain near "anchor"



"Needs*" Somewhere To Start

- ***"Allocating risk dollars back to WBS elements*"*** **- a.k.a. the "Needs" Method**
  - Offers a scheme for the "Variance Camp" to reduce budget overrun semi-variance when allocating
  - It uses PE as an "anchor" and distributes "risk dollars" to elements

NOT RECOMMENDED

$$b_k = pe_k + Risk\$ \sum_{i=1}^{n} \left(\frac{\rho_{ik} Need_i Need_k}{\sum_{j=1}^{n} \rho_{ij} Need_i Need_j} \right)$$

Where,
- C is the probability level of the total budget
- b_k is the allocated cost (budget line)
- pe_k is the initial estimate for element k
- $Risk\$ = F_T^{-1}(C) - pe_T$ is the total "at-risk" money to distribute
- $Need_k = F_k^{-1}(C) - pe_k$ when $F_k(pe_k) < C$; otherwise $Need_k = 0$
- ρ_{ij} is the correlation of elements i and j (full correlation matrix)
- $F^{-1}(C)$ is inverse distribution function (returns cost at %-tile C)

Definition for $Need_k$ in Question

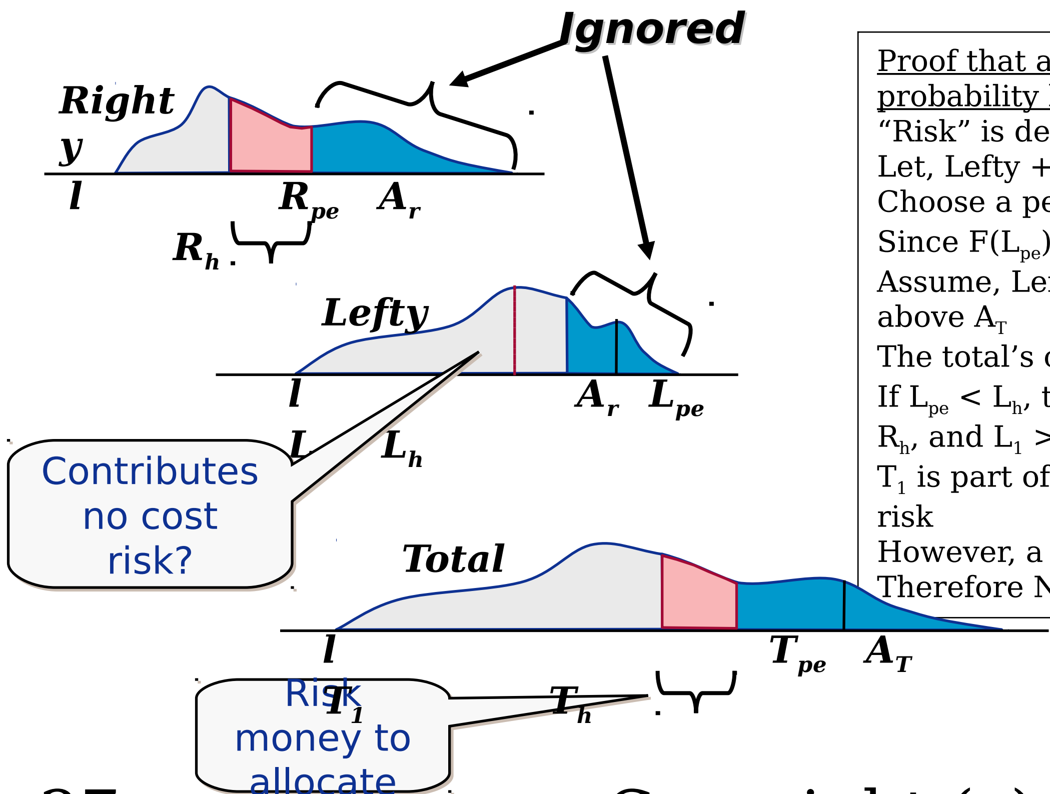
- **Issues with "Needs":**
  - Less risky *(left skew)* rows are subsidized, harming budgets for more risky *(right skew)* rows
  - Undo burden to rows with $Need > 0$ which can potentially send small items below their 0%-tile
  - $Need$ is analogous to semi-variance, yet an element's $Need$ changes when cost risk changes
  - At higher cost risk, costs lose their "bolstering" from associations with elements with $Need=0$
  - Method does not produce a solution at higher cost risk, when all elements' $Need=0$

- ***The problem is with what is being minimized***

* From "Allocating Risk Dollars Back to WBS Elements" Presentation, Stephen A. Book[4]

"Needs" Is A Changin'

- **Why is the equation "*Need$_k$ = 0*" so troublesome?**
- **In "Needs," only measures the range between PE and target percentile**
  - Ignores cost risk above the target percentile (thus most of the budget's cost risk)
  - In fact, an element's measure of contribution could go away completely
- **As long as our "anchor" doesn't move neither should our element's contribution**



<u>Proof that a row contributes to total's cost risk at any target probability level</u>
"Risk" is defined as the distribution above PE
Let, Lefty + Righty = Total
Choose a percentile C such that $F(L_{pe}) > C$ and $F(R_{pe}) < C$
Since $F(L_{pe}) > C$, thus $Need_L = 0$
Assume, Lefty's cost risk does not contribute to total's cost risk above A_T
The total's cost risk is defined in the range $[A_T, L_h + R_h]$
If $L_{pe} < L_h$, then some total value $T_1 > A_T$ exists where $T_1 = L_1 + R_h$, and $L_1 > L_p$,
T_1 is part of total's cost risk, thus L_1 contributes to total's cost risk
However, a Need ... is 0
Therefore $Need_L$ ...

<u>Proverb</u>

The only ~~constant~~ variance is change.

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Intro To "New Needs"

The "New Needs" Method…

Proverb

The ends justify using the means.

- **Apply Two Alterations**
  - Replace fluctuating ***Need*** with a constant measure, $\boldsymbol{v}$
  - Do not set ***Need*** ($\boldsymbol{v}$) to zero
- **Standard Deviation, $\boldsymbol{\sigma}$, Is Weak Measure of $\boldsymbol{v}$ Since It Is a Symmetrical Measure***
  - We want to estimate the BOSV, which lies to the right of the target budget
  - $\boldsymbol{\sigma}$ underestimates the consequence for elements whose distributions are skewed to the right
- **For $\boldsymbol{v}^2$, I recommend Using Positive Semi-Variance (PSV), $\boldsymbol{\sigma}_+^2$**
  - It is a constant measure that takes distribution skew into account & offers a rough BOSV metric
  - There are a number of ways to estimate PSV if you cannot calculate it directly

$$b_k = anchor_k + delta \frac{\sum_{j=1}^{n} \rho_{k,j} v_k v_j}{\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i,j} v_i v_j}$$

where,
$\boldsymbol{b_k}$ is the allocated cost (budget line)
$\boldsymbol{anchor_k}$ is anchor point for element $\boldsymbol{k}$
$\boldsymbol{delta}$ is the amount to distribute among elements
$\boldsymbol{\sigma_{+,k}}$ is the square root of the PSV
$\boldsymbol{\rho_{i,j}}$ is the (full) correlation of elements $\boldsymbol{i}$ and $\boldsymbol{j}$

$$v = \sigma_{+,k} = \sqrt{c_k \sum_{i=1}^{s} (x_i - \mu_k)^2}, \; for \; x_i > \mu_k$$

where,
$\boldsymbol{x_{i,k}}$ is a point in element $\boldsymbol{k}$'s random variable, $\boldsymbol{X_k}$
$\boldsymbol{s}$ is the number of data points in $\boldsymbol{X}$
$\boldsymbol{c_k}$ is the confidence level of the mean, $\boldsymbol{\mu_k}$

* Detailed in "Allocating Risk Dollars Back to WBS Elements" Presentation, Stephen A. Book[4]

Example Session

Six element example model with correlation*

Example Session						
WBS/CES	**Pt. Est.**	**PE %-tile**	**Mean**	**Std Dev**	**95%-tile**	**Semi-Variance**
Air Vehicle	$333,396	15%	$411,798	$74,435	$545,604	
Payload	$11,416	14%	$14,590	$3,006	$19,962	7,214,596
Propulsion	$16,271	17%	$20,496	$4,499	$28,744	17,007,376
Airframe	$112,250	49%	$116,277	$26,776	$165,003	593,555,769
Guidance	$186,979	15%	$251,304	$61,745	$366,670	3,327,328,489
IAT&C	$6,480	9%	$9,130	$2,163	$13,198	4,137,156

Correlation Matrix					
WBS/CES	Payload	Propulsion	Airframe	Guidance	I, A, T & C
Payload	1	0.32	0.32	0.21	0.33
Propulsion	0.32	1	0.25	0.17	0.15
Airframe	0.32	0.25	1	0.19	0.12
Guidance	0.21	0.17	0.19	1	0.18
I, A, T & C	0.33	0.15	0.12	0.18	1

We can estimate the semi-variance using a simple formula:

More ways to estimate at end of presentation

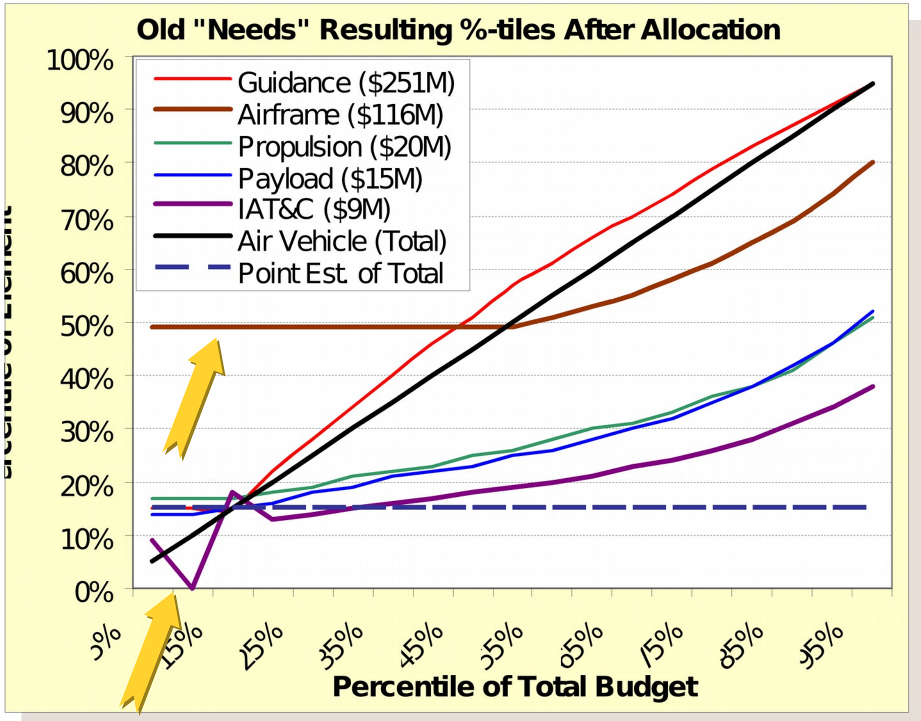
$$\sigma_+^2 \approx \frac{(high - \mu)^2}{4} \approx \frac{(cost_{95\%} - \mu)^2}{4}$$

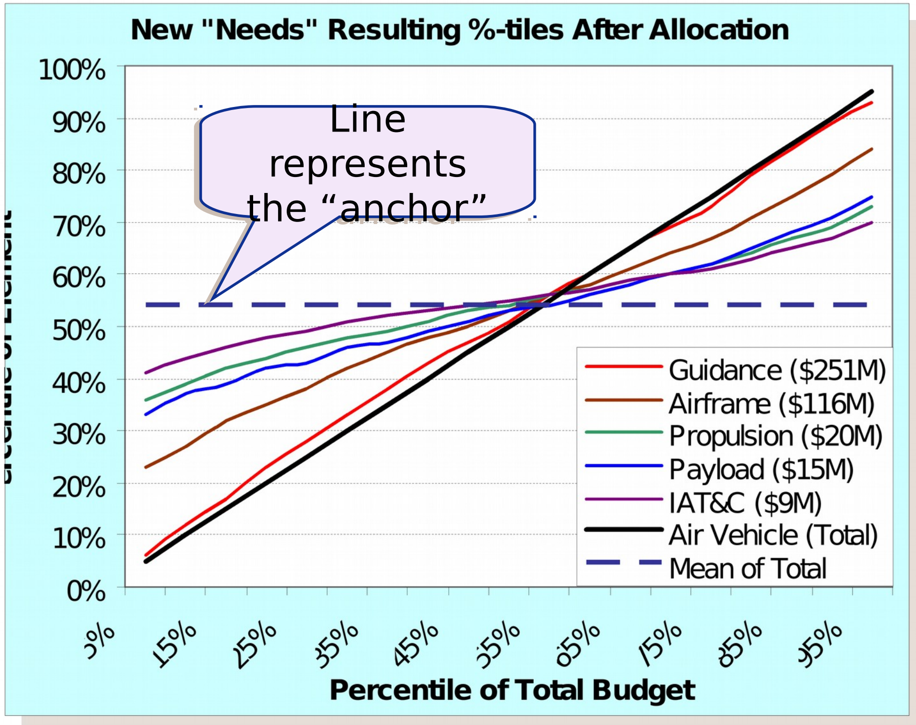
* "Air Vehicle Production Sub-WBS From AFCAA CRH Example[5]

"Needs" Comparison - Mean

- **Comparison of Resulting Percentiles After Allocation Performed**
  - Chart shows "New Needs" offers more stability than "Old Needs"
  - Anchoring at the mean offers "symmetry" for allocating at low and high percentiles

Scenario uses mean (μ) as anchor: $delta = budget_T - \mu_T$





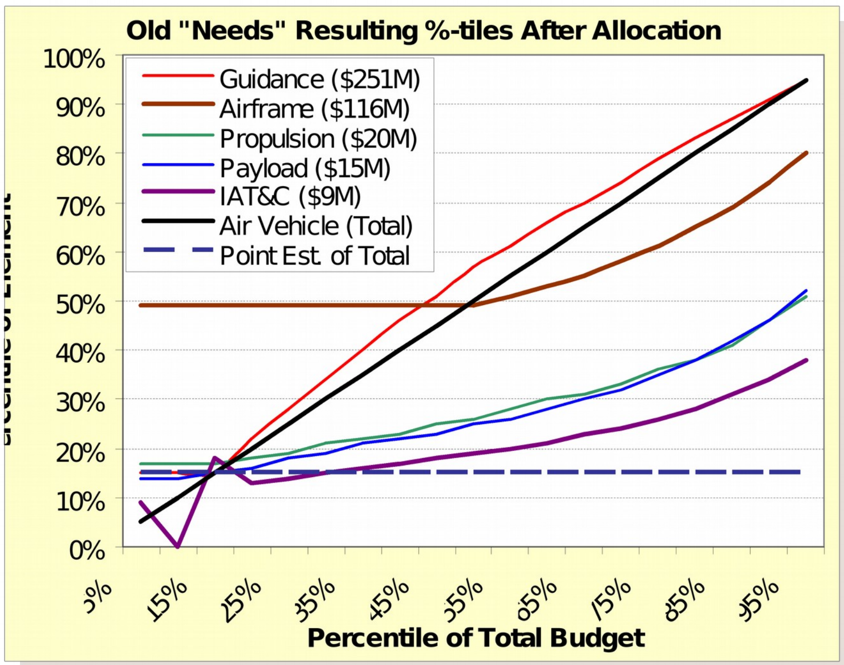
“Needs” Comparison - P.E.

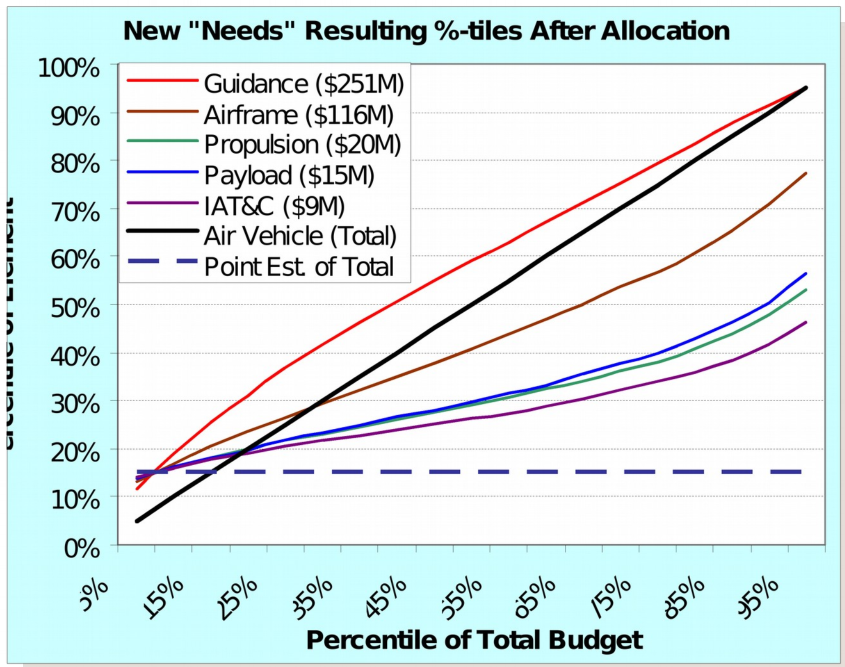
- **Alternate Comparison of Resulting Percentiles After Allocation**
  - “New Needs” supports alternatives to ***anchor*** and ***delta*** to suite your priorities
  - Some prefer to use point estimate instead of mean. The total’s $\boldsymbol{pe_T}$ is at 14%-tile.
  - We find the $\boldsymbol{cost_i}$ for each element at 14%-tile and use their sum them for to calc. ***delta***

Scenario uses PE %-tile as anchor:

$$anchor_i = F_i^{-1}\left(F\left(pe_{total}\right)\right) = \left[cost_i \ at \ \%tile \ of \ pe_{total}\right]$$

$$delta = budget_T - \sum_{i=1}^{n} anchor_i$$

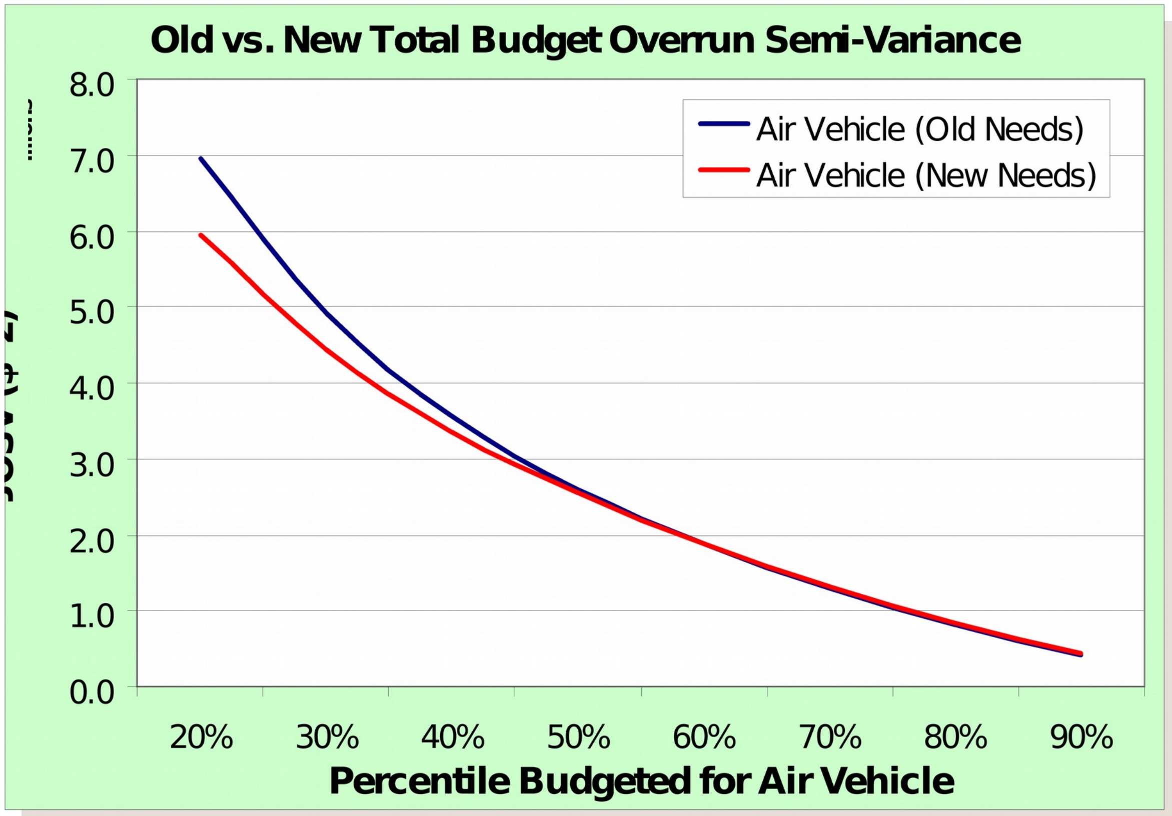




Minimization Performance

The ultimate goal of the “Variance Camp” is to minimize TBOSV

- **The chart below compares the TBOSV for “Old” and “New” methods**
  - The “New Needs” method is using the P.E. %-tile scenario from previous slide.
- **The new method outperforms the old for low confidences**
- **At high confidences, they are virtually identical**



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Conclusion

In Conclusion

- **Cost Risk allocation is a tool that serves a specific purpose**
  - Be sure that allocation serves your analysis goals
  - Only allocate when you have to encapsulate all money in WBS
  - Always allocate from where funds are managed
  - Allocate up or down the WBS
- **Two useful allocation methods were presented**
  - Consider the two camps of thought when picking a method
  - How to minimize the total average budget overrun (TABO)
  - How to (nearly) reduce the total budget overrun semi-variance (TBOSV)
    - Introduction to a more reliable “New Needs” method to replace old one
- **Round to stress the (lack of) precision of your numbers**
- **Be wary when discussing confidence levels after allocation**
  - This is a huge topic on its own!

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Questions?

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References:

[1] Information on the meanings of accuracy and precision:

http://en.wikipedia.org/wiki/Accuracy_and_precision

http://en.wikipedia.org/wiki/Significance_arithmetic#Uncertainty_and_error

[2] Examples of rounding at 2nd decimal of deviation:

http://physics.nist.gov/cuu/Constants/

Description of Semi-Variance

[3] "Selected Semi-Variance Estimators of Underreporting NonFarm Sole Proprietor Income," Chih-Chin Ho, Internal Revenue Service, IRC1996_028

http://www.amstat.org/sections/srms/proceedings/papers/1996_028.pdf

Detailed description of the "Needs" method and model

[4] "Allocating Risk Dollars Back to WBS Elements" Stephen A. Book, Chief Technical Officer, MCR, LLC SSCAG/EACE/SCAF Meeting 19-21 September 2006, also presented at SCEA Conference June 2006, DoDCAS Symposium February 2007

Additional information on uncertainty analysis and time-phased cost risk allocation

[5] "AFCAA Cost Risk Handbook" Alfred Smith et. al., CR-1254-3, 9 April

[6] "'Need' Needs Kneading" John Sandberg, Tecolote Research, Inc., presented at SSCAG Meeting 17 Jan 2007

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Helpful Formulae

General Reference

Here are general ways of calculating budget semi-variance, v^2

If you substitute μ for pe, you get the positive semi-variance, σ_+^2

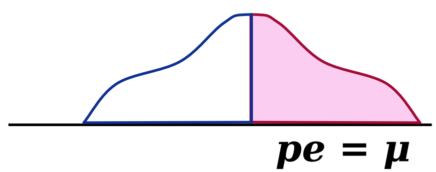
General Form

$$v^2 = c\sum_{i=1}^{s}(x_i - pe)^2,\quad for\ x_i > pe$$

where, x is a point in X, t is the # of points in X, and c is the prob. of overrun

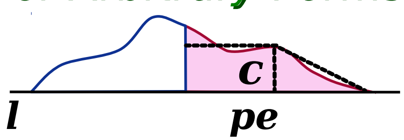
For Symmetrical Distribution Forms and pe = µ

$$v^2 = \frac{\sigma^2}{2}$$

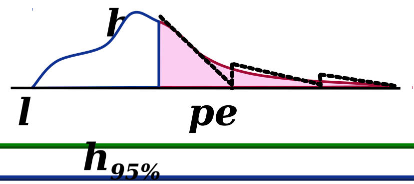


Rough Estimates For Arbitrary Forms

$$v^2 = \frac{c}{4}(h - pe)^2$$

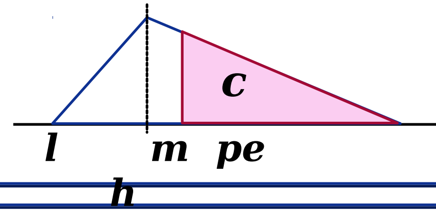


$$v^2 = \frac{c}{5}(h_{95\%} - pe)^2$$



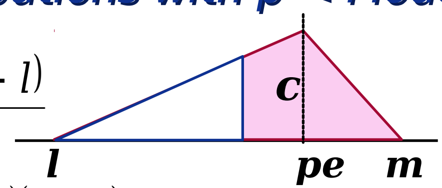
For Triangular Distributions with p ≥ Mode

$$v^2 = \frac{c}{6}(h - pe)^2$$



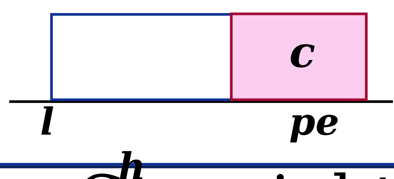
For Triangular Distributions with p < Mode

$$v^2 = \frac{(m-pe)^4}{6(h-l)^2} + \frac{2(m-c)^3(c-l)}{3(h-l)^2} + \left[\frac{h-m}{h-l}\right]\left[\frac{(h-m)^2}{2} + \frac{4(m-c)(h-m)}{3} + (m-c)^2\right]$$



For Uniform Distributions

$$v^2 = \frac{c}{3}(h - pe)^2$$



Probability Functions

General Reference

Cumulative density for triangular

$$1-c = F(x) = \begin{cases} 0, & x < l \\ \dfrac{(x-l)^2}{(\text{mode}-l)(h-l)}, & l \le x < \text{mode} \\ 1-\dfrac{(h-x)^2}{(h-\text{mode})(h-l)}, & \text{mode} \le x \le h \\ 1, & x > h \end{cases}$$

Probability density for triangular

$$f(x) = \begin{cases} 0, & x < l \\ \dfrac{2(x-l)}{(\text{mode}-l)(h-l)}, & l \le x < \text{mode} \\ \dfrac{2(h-x)}{(h-\text{mode})(h-l)}, & \text{mode} \le x \le h \\ 0, & x > h \end{cases}$$

Inverse CDF for triangular

$$F^{-1}(p) = \begin{cases} l + \sqrt{p(\text{mode}-l)(h-l)}, & p < F(\text{mode}) \\ h - \sqrt{(1-p)(h-\text{mode})(h-l)}, & F(\text{mode}) \le p \end{cases}$$

Mean and variance for triangular

$$\mu = \frac{(l + \text{mode} + h)}{3}$$

$$\sigma^2 = \frac{(\text{mode}-l)(\text{mode}-h) + (h-l)^2}{18}$$

Cumulative density for uniform

$$1-c = F(x) = \begin{cases} 0, & x < l \\ \dfrac{(x-l)}{(h-l)}, & l \le x \le h \\ 1, & x > h \end{cases}$$

Probability density for uniform

$$f(x) = \begin{cases} 0, & x < l \\ \dfrac{1}{(h-l)}, & l \le x \le h \\ 0, & x > h \end{cases}$$

Inverse CDF for uniform

$$F^{-1}(p) = l + p(h-l)$$

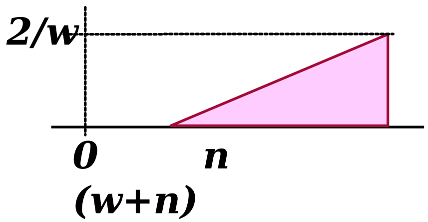
Mean and variance for uniform

$$\mu = \frac{(l+h)}{3} \qquad \sigma^2 = \frac{(h-l)^2}{12}$$

Derivations for Estimates

General Reference

“cost-distance-squared” of displaced triangle



$$v^2 = \int_0^w (x+n)^2 \left(\frac{2x}{w^2}\right) dx$$

$$v^2 = \frac{2}{w^2}\int_0^w \left[x^3 + 2nx^2 + n^2 x\right] dx$$

$$v^2 = \frac{2}{w^2}\left(\frac{w^4}{4} + \frac{2nw^3}{3} + \frac{n^2w^2}{2}\right)$$

$$v^2 = \frac{w^2}{2} + \frac{4nw}{3} + n^2$$

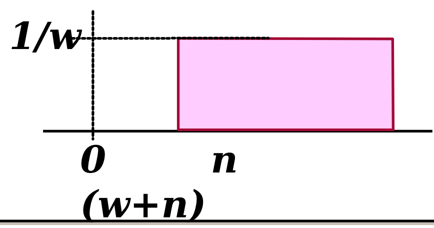
“cost-distance-squared” of displaced rectangle

$$v^2 = \int_0^w (x+n)^2 \left(\frac{1}{w}\right) dx$$

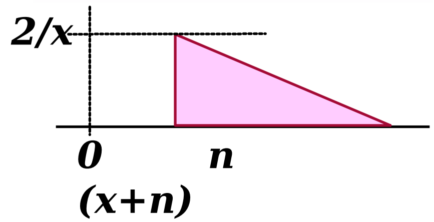
$$v^2 = \frac{1}{w}\int_0^w \left(x^2 + 2nx + n^2\right) dx$$

$$v^2 = \frac{1}{w}\left(\frac{1}{3}w^3 + nw^2 + n^2 w\right)$$

$$v^2 = \frac{w^2}{3} + nw + n^2$$



“cost-distance-squared” of displaced triangle



$$v^2 = \int_0^w (x+n)^2 \left(\frac{2(w-x)}{w^2}\right) dx$$

$$v^2 = \frac{2}{w^2}\int_0^w \left[\left(x^2 + 2nx + n^2\right)(w-x)\right] dx$$

$$v^2 = \frac{2}{w^2}\int_0^w \left[wx^2 - x^3 + 2nwx - 2nx^2 + n^2w - n^2x\right] dx$$

$$v^2 = \frac{2}{w^3}\left(\frac{w^4}{3} - \frac{w^4}{4} + nw^3 - \frac{2nw^3}{3} + n^2w^2 - \frac{n^2w^2}{2}\right)$$

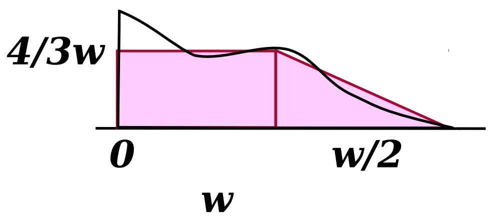
$$v = 2\left(w^2\left(\frac{1}{3} - \frac{1}{4}\right) + w\left(n - \frac{2n}{3}\right) + \left(n^2 - \frac{n^2}{2}\right)\right)$$

$$v^2 = \frac{w^2}{6} + \frac{2nw}{3} + n^2$$

Derivations for Estimates

General Reference

Rough estimate for curve tapering down (fat tail)



$$v^2 = area_1 v_1^2 + area_2 v_2^2$$

$$v^2 = \frac{2}{3}\left[\frac{(w/2)^2}{3}\right] + \frac{1}{3}\left[\frac{(w/2)^2}{3} + (w/2)(w/2) + (w/2)^2\right]$$

$$v^2 = w^2\left[\frac{2}{4(9)} + \frac{1}{4(9)} + \frac{1}{12} + \frac{1}{12}\right] = w^2\frac{9}{36}$$

$$v^2 = \frac{w^2}{4}$$

$$v^2 = area_1 v_1^2 + area_2 v_2^2 + area_3 v_3^2$$

$$v^2 = \frac{9}{13}\left[\frac{(w/3)^2}{6}\right] + \frac{3}{13}\left[\frac{(w/3)^2}{6} + \frac{2}{3}(w/3)(w/3) + (w/3)^2\right]$$

$$+ \frac{1}{13}\left[\frac{(w/3)^2}{6} + \frac{2}{3}(w/3)(2w/3) + (2w/3)^2\right]$$

$$v^2 = \frac{w^2}{13}\left[\frac{9(9)}{6(9)} + \frac{3}{6(9)} + \frac{6}{3(9)} + \frac{3}{9} + \frac{1}{6(9)} + \frac{4}{3(9)} + \frac{4}{9}\right] = w^2\frac{147}{13(54)}$$

$$v^2 = \frac{49}{234}w^2 \approx \frac{w^2}{5}$$

Rough estimate for curve tapering up (thin tail)